

atom Sigma material increase achieved electron beam electron volt Beta function coherent fraction
photon energy radiation energy ray photon shown electric field
order energy undulator photon emittance time
example photon emittance total emittance
change function approximately divided rings section storage number obtain magnitude detail beam experiment Delta source size
electron emittance
phase space seen picometre radians square root video equals third generation due begin

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Video



Contents and objectives of this video



- “Single-electron”, or photon, emittance
- Combining electron and photon emittances
- The diffraction limit
- Factors determining emittance

In this penultimate video of the second of three sections this third week, we look at how to combine the electron emittance from an ensemble of electrons making up the electron beam in a storage ring, with the single-electron emittance or photon emittance. We see that the latter is, for a given photon energy, a fixed quantity. This video is quite detailed, so you can get by without understanding all of the details. But it nonetheless provides insights if you ever want to build your own beamline and need to decide on the optimal source for the energy range that you'll be working in.

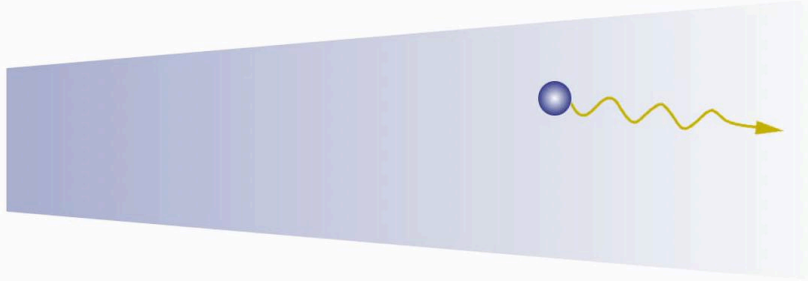
Notes

Summary



0m 05s

Electron and photon emittances



Also called "single-electron" emittance

An electron beam has a distribution of individual positions and angles for each contributing electron, as we discussed in the previous video. But for a given electron emitting a photon, are the direction and origin of that photon perfectly defined? The answer is no, due to diffraction effects. This photon emittance is also called the single-electron emittance, as I've already mentioned.

Notes


Summary



0m 45s

Electron and photon emittances



$$\epsilon_{x,y} = \sigma_{x,y} \sigma'_{x,y} = \left[(\sigma_{x,y}^e)^2 + (\sigma^p)^2 \right]^{1/2} \left[(\sigma'_{x,y})^2 + (\sigma'^p)^2 \right]^{1/2}$$


e-beam emittance

Function of storage-ring design
Constant around ring
Different for x- and y-planes

photon emittance

Function only of photon energy
Consequence of diffraction
Equal in x- and y-planes

$$\sigma^p = \frac{\sqrt{\lambda L}}{4\pi}$$

$$\sigma'^p = \sqrt{\frac{\lambda}{L}}$$

(L = undulator length)

See also explanation of convolutions in the supplementary text "Fourier Transforms and Convolutions Made Simple"

The total emittance of the emitted synchrotron radiation therefore depends both on the quality or emittance of the electron beam, plus the contribution from the photon emittance. From here on in, we use the colour code that the electron emittance is in blue, the photon emittance in gold and the total emittance in green. The total emittance is the convolution of the electron and photon emittances. The convolution operation between two functions is given by the circled times sign shown here. Mathematically, the total emittance in either the X or the Y direction is the product of the total source size times the total divergence in that direction. These, in turn, are equal to the square roots of the sum of the squares of each contribution, just as the hypotenuse of a triangle is equal to the square root of the sum of the squares of the two right-angle sides. The electron emittance is a function of the storage ring design. In DLSRs, the goal has been to reduce the electron emittance by approximately two orders of magnitude. The electron emittance is a constant around the ring. In general, the horizontal electron emittance is larger than the vertical electron emittance.

Notes

Summary



1m 15s

Electron and photon emittances



$$\epsilon_{x,y} = \sigma_{x,y} \sigma'_{x,y} = \left[(\sigma_{x,y}^e)^2 + (\sigma^p)^2 \right]^{1/2} \left[(\sigma'_{x,y})^2 + (\sigma'^p)^2 \right]^{1/2}$$

e-beam emittance

Function of storage-ring design
Constant around ring
Different for x- and y-planes

photon emittance

Function only of photon energy
Consequence of diffraction
Equal in x- and y-planes

$$\left. \begin{aligned} \sigma^p &= \frac{\sqrt{\lambda L}}{4\pi} \\ \sigma'^p &= \sqrt{\frac{\lambda}{L}} \end{aligned} \right\} \left. \begin{aligned} \epsilon^p &= \frac{\lambda}{4\pi} \\ \beta^p &= \frac{L}{4\pi} \end{aligned} \right\}$$

(L = undulator length)

See also explanation of convolutions in the supplementary text "Fourier Transforms and Convolutions Made Simple"

$$\epsilon^p = \frac{98.66}{E[\text{keV}]} [\text{pm rad}]$$

As has already been stressed, the photon emittance depends only on the photon energy or wavelength. The effective photon source size, σ^p , for photons produced in an undulator, the insertion device, which we will encounter in detail in next week's videos, is equal to the square root of the photon wavelength, λ , times the undulator length, L , divided by four π . The same photon's effective divergence, σ'^p , is the square root of λ upon L . Multiplying these together yields the photon emittance, which is simply ϵ^p is equal to λ divided by four π . We can re-express this in terms of the photon energy to obtain the practical expression that ϵ^p , in picometre radians, is 98.66 divided by E , if that is given in kiloelectron volts. So a 10 kiloelectron volt photon has a natural photon emittance of approximately 10 picometre radians. The ratio of σ^p to σ'^p , gives the photon Beta function, which is L divided by four π . Note that this is independent of the photon energy and depends only on the undulator length.

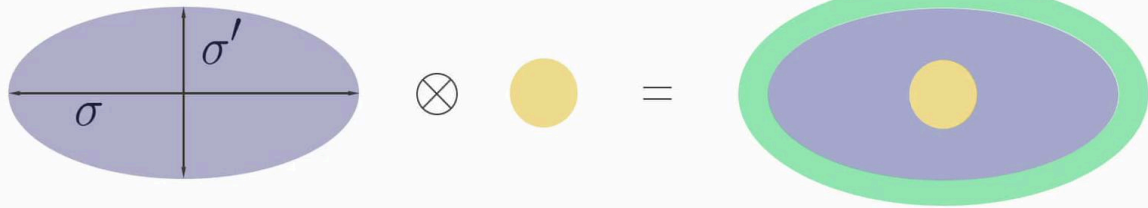
Notes

Summary



2m 40s

Electron and photon emittances



$$\epsilon_{x,y} = \sigma_{x,y} \sigma'_{x,y} = \left[(\sigma_{x,y}^e)^2 + (\sigma^p)^2 \right]^{1/2} \left[(\sigma'_{x,y})^2 + (\sigma'^p)^2 \right]^{1/2}$$

e-beam emittance

Function of storage-ring design
Constant around ring
Different for x- and y-planes

photon emittance

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Consequence of diffraction
Equal in x- and y-planes

$$\epsilon^p = \frac{98.66}{E[\text{keV}]} [\text{pm rad}]$$

$$\left. \begin{array}{l} \sigma^p = \frac{\sqrt{\lambda L}}{4\pi} \\ \sigma'^p = \sqrt{\frac{\lambda}{L}} \end{array} \right\} \begin{array}{l} \epsilon^p = \frac{\lambda}{4\pi} \\ \beta^p = \frac{L}{4\pi} \end{array}$$

(L = undulator length)

See also explanation of convolutions in the supplementary text "Fourier Transforms and Convolutions Made Simple"

Alternatively, we can represent the convolutions of the emittance contributions in phase space, which we'll see is a profitable exercise. In this depiction, the convolution is more intuitively illustrated. Roughly speaking, one can interpret the green ellipse being the result of drawing the blue ellipse with a marker pen with a nib the shape of the yellow ellipse. Take a closer look at the supplementary material "Fourier Transforms and Convolutions Made Simple".

Notes

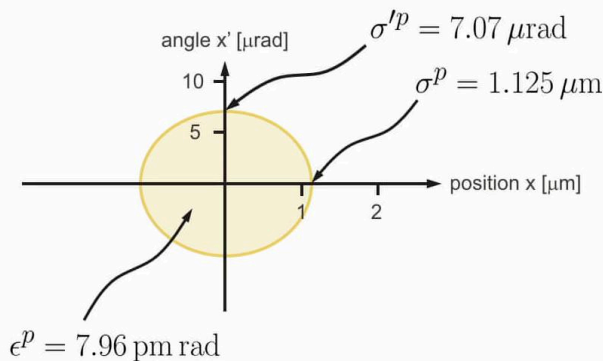
Summary



4m 06s

Matching electron and photon emittances

Hard x-ray photons



1-Å (12.4 keV) radiation
L = 2 m undulator
Valid for both x- and y-planes

Let's pursue this further, considering the four cases of soft and hard x-rays at both third and fourth generation synchrotrons. We begin with the generation of 10 kiloelectron volt photons at a third generation beamline served by a two metre undulator. We can immediately draw the yellow photon emittance ellipse, according to the equations for the photon source size and the divergence that we have just been introduced to. From these, we obtain Sigma P is equal to 1.125 microns and Sigma P prime is equal to 7.07 microradians. The ellipse area is proportional to the emittance, which is 7.96 picometre radians. The photon Beta function, L divided by four Pi, is 0.159 metres.

Notes

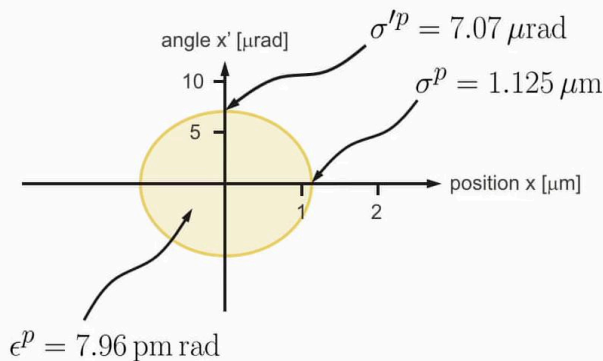
Summary



4m 40s

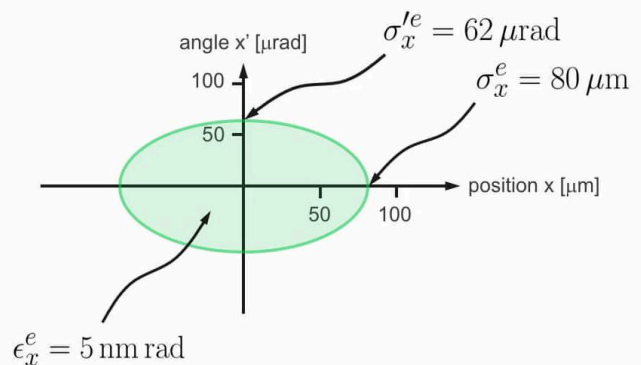
Matching electron and photon emittances

Hard x-ray photons



1-Å (12.4 keV) radiation
L = 2 m undulator
Valid for both x - and y -planes

Electrons, x -plane,
3rd generation facility



Undulator straight
 $\beta = 1.28 \text{ m}$

We then draw on the right the electron emittance ellipse in blue for a storage ring with a five nanometre electron emittance which, in this example, we have divided up into a source size of 80 microns and a divergence of 62 microradians. The Beta function is thus 1.28 metres. We now superimpose the photon emittance on top of the electron emittance, using the same scales for Sigma and Sigma prime. Did you spot it? It's here. As you can see, it's really tiny compared to the electron emittance, and the resulting total emittance ellipse, in green, hardly differs at all from the electron emittance ellipse. In other words, the total emittance depends almost entirely on the electron emittance for hard x-rays generated at a third generation facility.

Notes

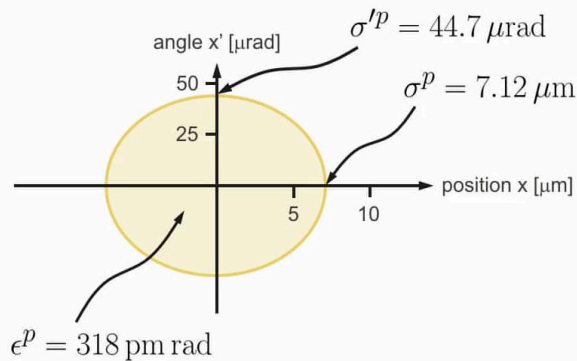
Summary



5m 45s

Matching electron and photon emittances

Soft x-ray photons



4-nm (310 eV) radiation
L = 2 m undulator
Valid for both x- and y-planes

Now, let's go through the same exercise for four nanometre, or approximately 300 electron volt, soft x-rays, also produced by a two metre undulator at a third generation facility, with the same electron emittance and Beta functions as given before for the hard x-rays. The photon phase space ellipse is significantly larger than that for the hard x rays, as the wavelength is 40 times larger. Indeed, looking back at our equations for the photon source size and divergence, we see that it is proportional to the square root of Lambda, so we expect the ellipse to grow by the square root of 40, or about 6.3, in both directions, compared to the one angstrom photons. This yields a photon source size of 7.12 microns, and a divergence of 44.7 microradians, a photon emittance of 318 picometre radians and the same Beta function of 0.159 metres.

Notes

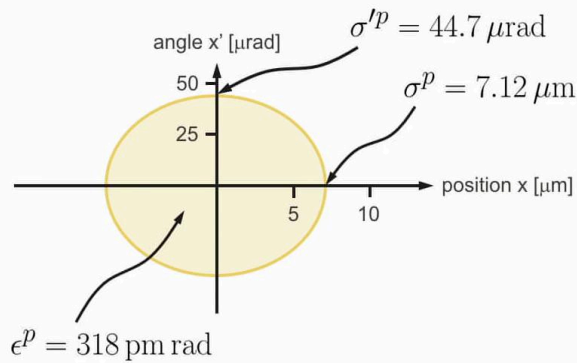
Summary



6m 45s

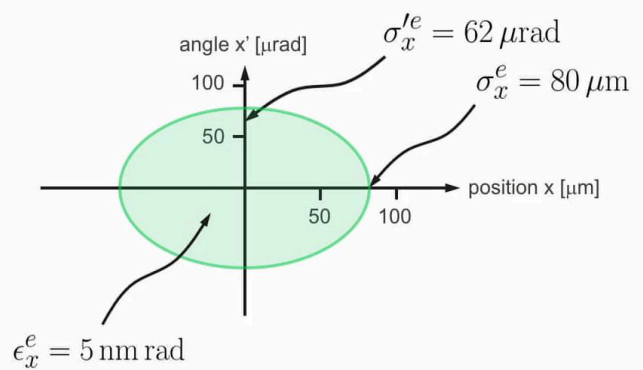
Matching electron and photon emittances

Soft x-ray photons



4-nm (310 eV) radiation
L = 2 m undulator
Valid for both x- and y-planes

Electrons, x-plane,
3rd generation facility



Undulator straight
 $\beta = 1.28$ m

The electron phase space ellipse remains unchanged, as we are considering the same third generation facility and an undulator with the same length. But now, if we superimpose the photon ellipse on top of the electron ellipse, we see that the former, although still smaller, isn't teensy, teensy, weensy like it was before in the case of hard x-rays. And indeed, with regards to the divergence, has a fairly similar value. The result is that the convolution, in other words, the total emittance, is increased somewhat, most obviously in the divergence.

Notes

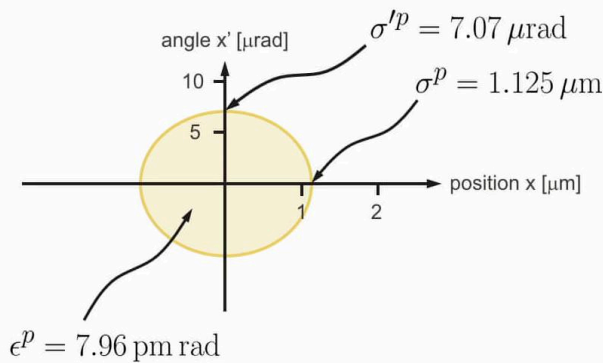
Summary



7m 54s

Matching electron and photon emittances

Hard x-ray photons



1-Å (12.4 keV) radiation
L = 2 m undulator
Valid for both x- and y-planes

We now go through the exact same procedure, but now for a fourth generation facility, for which we assume an electron emittance of 150 picometre radians, some 33 times smaller than for the third generation storage example we've just looked at. The photon phase space ellipse for hard x-rays remains as it was for the third generation case.

Notes

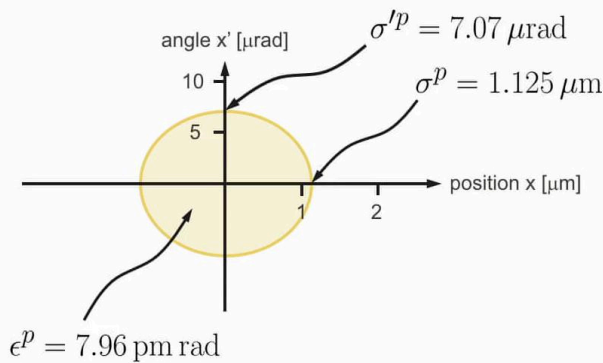
Summary



8m 32s

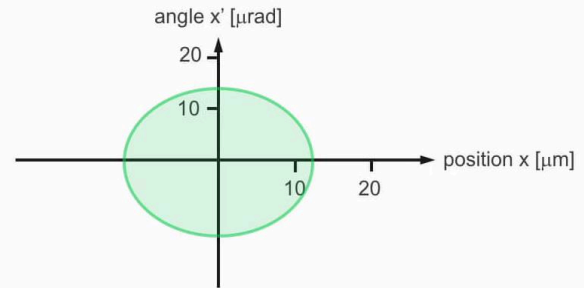
Matching electron and photon emittances

Hard x-ray photons



1-Å (12.4 keV) radiation
L = 2 m undulator
Valid for both x- and y-planes

Electrons, x-plane,
4th generation facility



Undulator straight
 $\beta = 1.00 \text{ m}$

The electron emittance is divided up into a source size of 12.25 microns and a divergence of 12.25 microradians. The electron beam Beta function is therefore one metre. Now, the photon phase space ellipse, while still considerably smaller than that for the electrons, is, relatively speaking, much larger than at the third generation ring. The total emittance is moderately increased relative to the electron emittance.

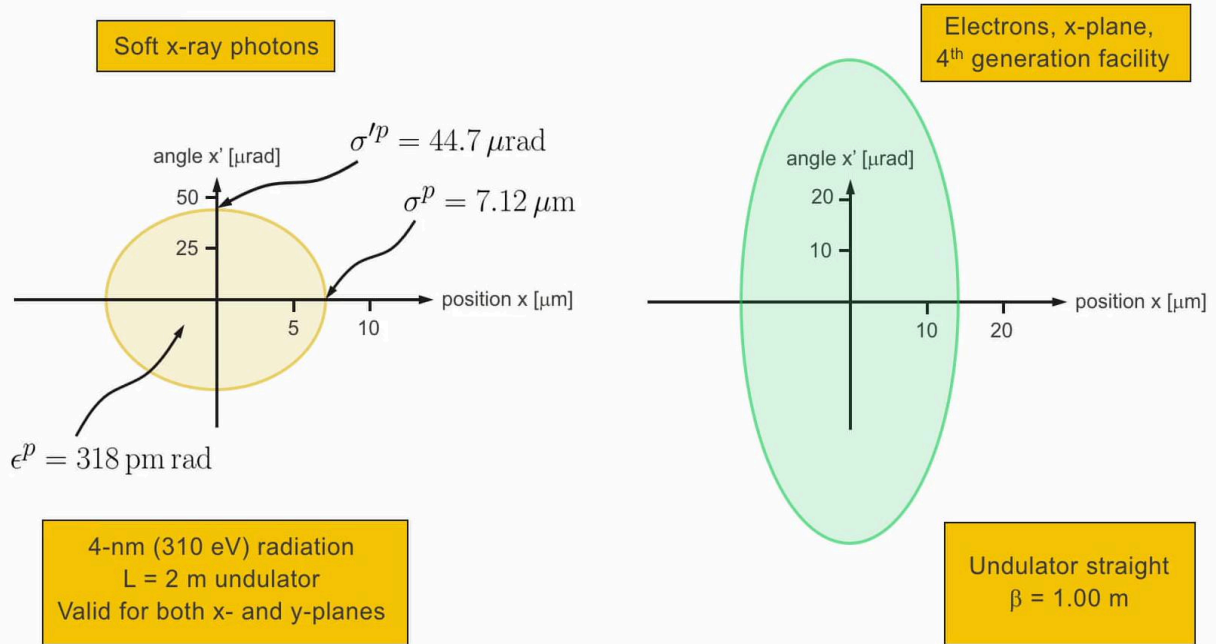
Notes

Summary



9m 00s

Matching electron and photon emittances



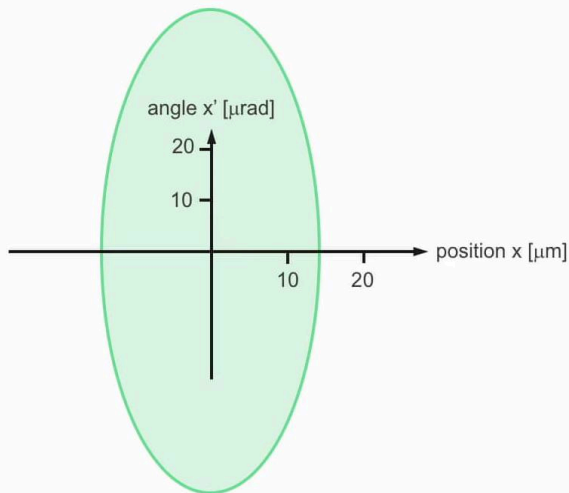
Lastly, we consider the four nanometre radiation when superimposed on the electron emittance, which is seen to be significantly larger with regards to the divergence and about half the source size. The total emittance is therefore considerably larger than both the electron and photon emittances.

Notes

Summary



Matching electron and photon emittances



Now, what is immediately evident from this last example is that we have managed to pick the worst of both worlds. The Beta function of the electron emittance means that, compared to the fixed photon Beta function, for a given undulator length, that is, the source size is too large and too collimated, so that the total emittance both has a large source size and is quite divergent.

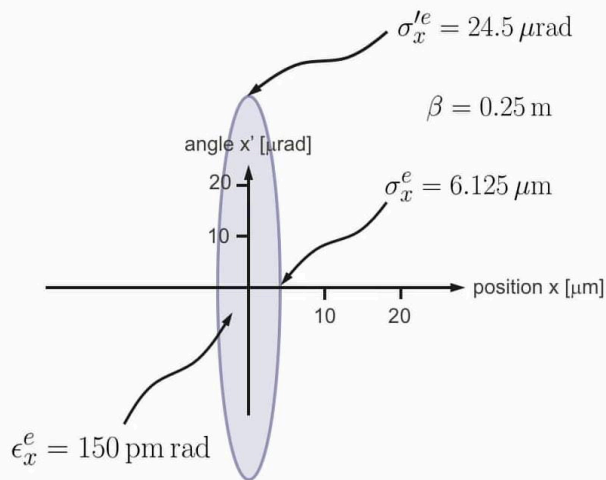
Notes

Summary



9m 56s

Matching electron and photon emittances



If we play with the electron optics, however, we can manipulate the electron Beta function so that, for the same electron emittance, here, 150 picometre radians, it has the same aspect ratio, or emittance, as the photon ellipse, thereby minimising the total emittance.

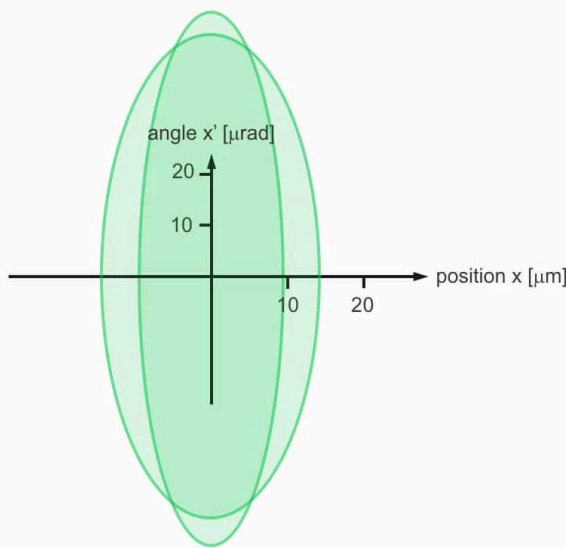
Notes

Summary

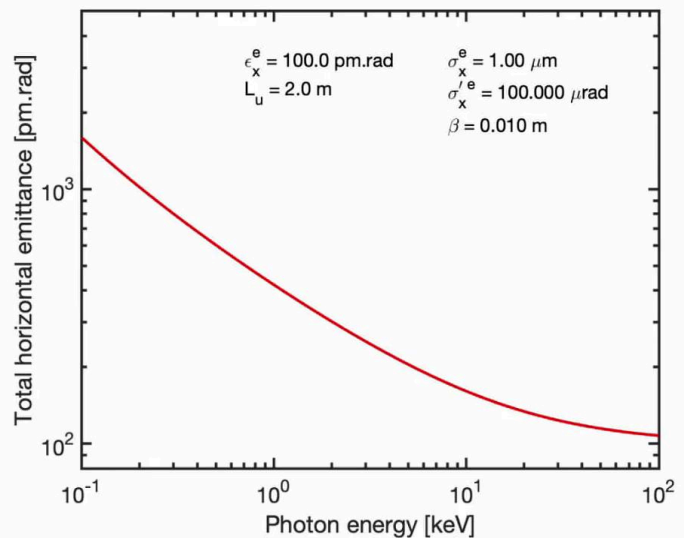
10m 23s



Matching electron and photon emittances



Best ph-e match when
 $\beta = L/4\pi$ (same as β^p)



Which we now compare with the total emittance we got before for the unoptimised Beta function. We see it's considerably larger here. So in summary, we can play with the electron beam, so its phase space matches where it is important that it matches the photon emittance by having a Beta function of L upon four π . We can see how this has an impact on the total emittance as a function of photon energy in the following animation. We assume a 100 picometre radion electron emittance, and change the Beta function from one centimetre to 40 metres. In other words, from a tiny and very divergent source to a large and very well-collimated source. Now, before we run the animation, we can predict two things. First, the changes will be much more modest at low photon energies, where the photon emittance anyway dominates. And secondly, we should expect the lowest emittance, across the board, when Beta is equal to L upon four π , which, for a two metre undulator, is at a Beta of 0.159 metres. Feel free to rewind the video to play this again and again. By the way, don't forget that this, and all the other animations, are downloadable from the MOOC website as supplementary material. Anyway. So let's go. We see it's gone through that minimum there. And now it's getting larger and larger and larger.

Notes

Summary



10m 43s

The diffraction limit

- As electron-beam emittance reduces, photon-beam emittance begins to dominate
- Diffraction limit to SR defined when $\varepsilon_x^e = \varepsilon_p$

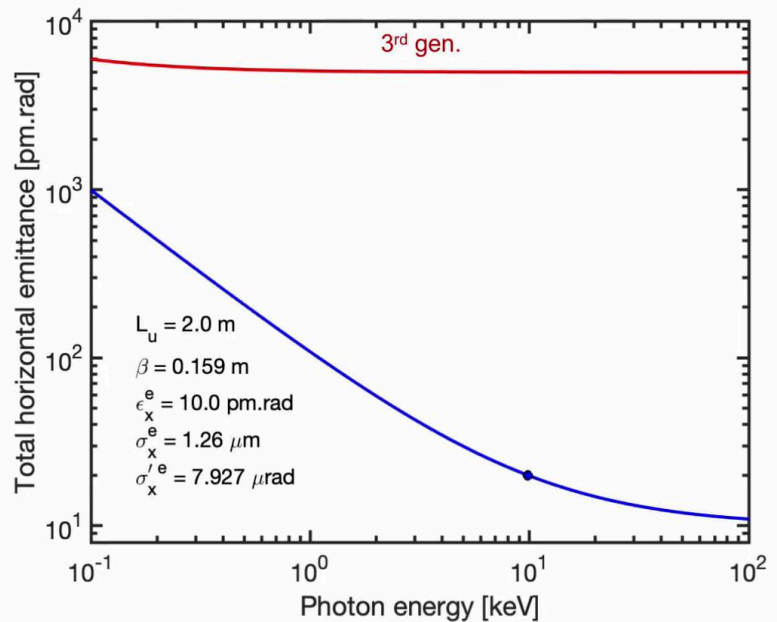


$$\epsilon^p = \frac{\lambda}{4\pi}$$

$$\lambda_{DL} = 4\pi\epsilon^p$$

- Diffraction-limited photon energy defined as

$$h\nu_{DL}[\text{keV}] = 1239.8/4\pi(\epsilon^p[\text{pm rad}])$$
- Once $h\nu_{DL}$ reached, further reduction in ε_x^e brings little benefit



So we have seen two examples of third and fourth generation facilities with total electron emittances of five nanometre radians and 150 picometre radians, respectively. In the former case, the photon emittance only has an impact on the total at low photon energies, which we can see in the modest upswing in total emittance for energies below approximately one kiloelectron volt in the graph on the right. The diffraction limit is defined as being achieved when the electron beam has the same emittance and Beta function as the photon emittance. This happens when the diffraction limited wavelength, λ_{DL} , is equal to four Pi Epsilon P, or when the photon energy, in kiloelectron volts, is equal to 1239.8 divided by four Pi Epsilon P, when expressed in picometre radians. The progress to a DLSR from five nanometre radians down to 10 picometre radians, is shown in the animation on the right. 10 picometre radians is over an order of magnitude lower than the present best-case DLSR in the upgraded ESRF EBS in Grenoble. But plans are underfoot to achieve such values at, for example, PETRA IV in Hamburg. The blue dot represents a diffraction-limited photon energy, which has a total emittance exactly twice the value of the electron emittance.

Notes

Summary



12m 37s

The diffraction limit

- As electron-beam emittance reduces, photon-beam emittance begins to dominate
- Diffraction limit to SR defined when $\varepsilon_x^e = \varepsilon_p$

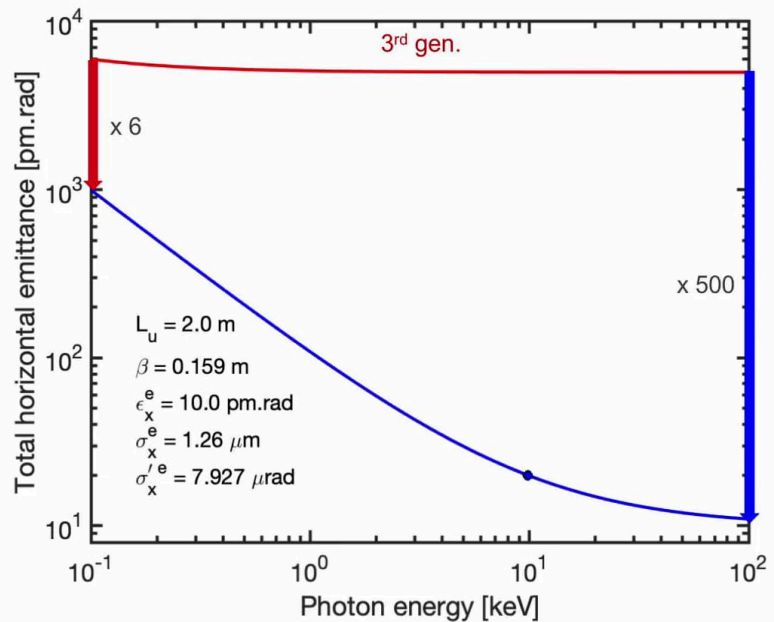


$$\epsilon^p = \frac{\lambda}{4\pi}$$

$$\lambda_{DL} = 4\pi\epsilon^p$$

- Diffraction-limited photon energy defined as

$$h\nu_{DL}[\text{keV}] = 1239.8/4\pi(\epsilon^p[\text{pm rad}])$$
- Once $h\nu_{DL}$ reached, further reduction in ε_x^e brings little benefit



This is not surprising, as the electron and photon phase space ellipses at the diffraction-limited energy are identical, and thus each of these contribute equally to the total emittance. The gains made increase strongly with photon energy. We see that the factor improvement in emittance at 100 electron volts is only about six, while at 100 kiloelectron volt, it's 500.

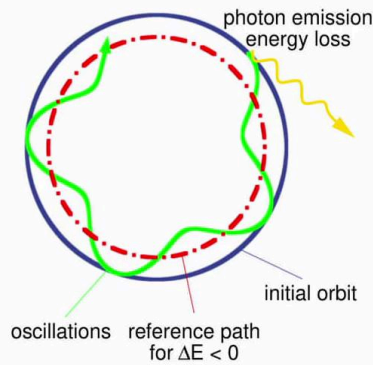
Notes

Summary

14m 20s



Factors determining emittance - radiation equilibrium

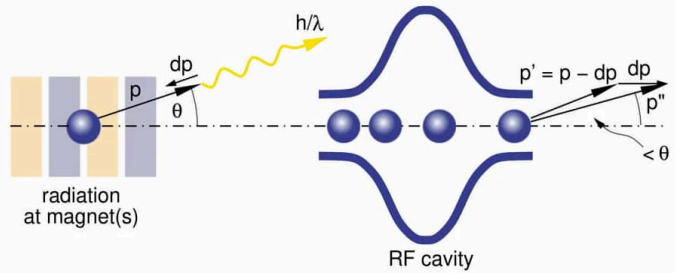


MAX IV (DLSR)
minimizes this

Quantum excitation

Increases electron dispersion
Broadens beam
Increases emittance

V



Radiation damping

NSLS II
maximizes this

Reduces angular spread
Decreases emittance

Now, what determines the electron emittance? It turns out that it is determined by a so-called radiation equilibrium, a balance between competing processes, namely quantum excitation, which is bad, and radiation damping, which is good. At the NSLS II in Brookhaven, the machine performance is optimised by maximising radiation damping, while in the next generation of DLSRs, such as MAX IV, quantum excitation has been minimised. When an electron emits a photon, it loses the energy of that photon. This causes it to oscillate around a new reference orbit, thus broadening the beam and thereby increasing the emittance. Moreover, the energy dispersion of the electron beam will increase. This quantum excitation can be reduced by designing the magnet lattice so that the electron's energy dispersion is minimised at the main locations of radiation, namely, the bending magnets. This is achieved by horizontal focussing at the bends and the use of many small deflection angle bends, in so-called multi-bend achromat lattices, to limit dispersion growth. We discuss this next week.

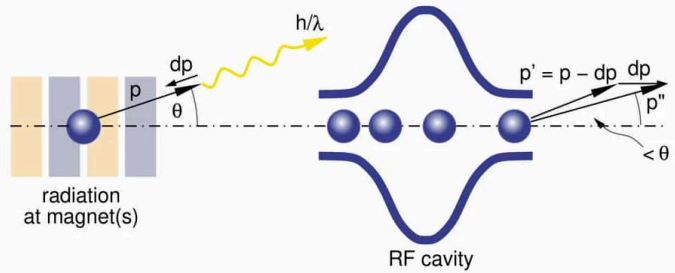
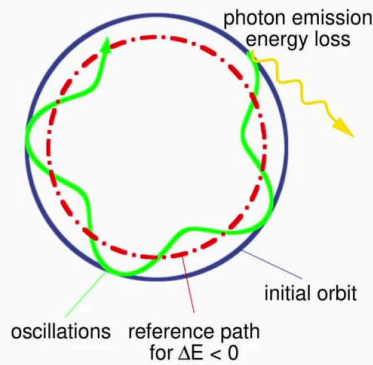
Notes

Summary



14m 50s

Factors determining emittance - radiation equilibrium



MAX IV (DLSR)
minimizes this

Quantum excitation

Increases electron dispersion
Broadens beam
Increases emittance

V

Radiation damping

Reduces angular spread
Decreases emittance

NSLS II
maximizes this

Radiation damping describes how axial acceleration by the RF cavity to replenish the electron's energy reduces the angular deviation of the electron from the ideal orbit, by adding momentum, DP , along the central axis, which had previously been lost at some angle, θ , to the central axis. This replenishment of the lost momentum on axis thus tilts the total momentum to shallower angles, and thereby reduces the electron divergence. Radiation damping therefore minimises the transverse momenta of the electrons. By introducing high-field bending magnets and, or very high power damping wigglers, radiation damping can be maximised.

Notes

Summary



In the next video...



In the last video of this section, we'll discover the property of coherence. Synchrotrons are famously, or infamously, incoherent light sources, in contrast to conventional lasers. Nonetheless, they do have a small coherent fraction that is exploited in many experiments. DLSRs provide an increase in coherent fraction close to the increase in brilliance, reaching several percent in the hard x-ray regime. This promises to be a game-changing improvement in many fields, most notably in x-ray imaging and branches of crystallography.

Notes

Summary



17m 21s